ANALYSIS OF ENERGY AND SPECTRAL CHARACTERISTICS OF A GASDYNAMIC LASER WITH N_2–DCl MIXING

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The use of hydrogen halide molecules in a mixture with diatomic homonuclear molecules (H_2, D_2, N_2) to achieve lasing in gasdynamic lasers (GDLs) has been discussed for a rather long time in the literature [1-3]. A N_2–DCl mixture is one of the possible working media which, according to the data of [3], have high specific energy characteristics. Using the harmonic oscillator model to calculate the energy characteristics of a GDL based on this mixture [4], we demonstrated the feasibility of obtaining coherent radiation with $\lambda \simeq 5.3$ µm and a specific energy 5 to 10 times the corresponding values for CO gasdynamic lasers, which have a similar lasing wavelength and are being studied extensively. An even higher energy conversion efficiency can be obtained in lasers with DCl mixed into a supersonic flow of preheated molecular gas. Calculations [3, 4], however, do not provide detailed information about the inverse and energy characteristics of such systems, since the actual distribution of DCl molecules among the vibrational levels during expansion in supersonic nozzles differs substantially from the Boltzmann distribution and lasing occurs in several rotation–vibration transitions (multiple-frequency lasing) [5].

Our aim here is to make a detailed analysis of the energy and spectral characteristics of a GDL in which DCl molecules are excited in a VV' exchange with N_2 molecules when mixed in a supersonic flow.

In describing the kinetics of vibrational energy exchange and radiative transitions we model the vibrations of N_2 and DCl molecules with a Morse oscillator and assume that the rotational degrees of freedom are in equilibrium with the translational degrees of freedom. The equations for the relative populations of excited states with V = n in DCl and N_2 in this case have the form

$$\frac{dX_n(i)}{dt} = Q_V^n(i),\quad Q_V^n(i) = q_{VT}^n(i) + q_{VV}^n(i) + q_{VV'}^n(i) + q_I^n(i)\,\delta_{i,1}. \tag{1}$$

Here $X_n(i) = N_{n,i}/N_i$; N_{ni} (i = 1 for DCl, i = 2 for N_2) is the density of molecules of the i-th species, excited to the vibrational state with V = n; q_{VT}^n, q_{VV}^n, $q_{VV'}^n$, and q_I^n are terms that characterize the change in the number of molecules in the V = n state as a result of the vibration–translation VT, intramode VV and intermode VV' rotation–rotation exchange, and induced transitions, respectively; and $\delta_{i,1}$ is the Kronecker symbol.

The parameters of the flow in the nozzle and inside the cavity are calculated in the approximation of one-dimensional flow of a nonviscous non-heat-conducting gas, and the molecular mixing is assumed to be instantaneous, i.e., we assume that the mixing zone is the surface of discontinuity of the macroscopic parameters while the values of X_n do not change in the passage through this surface. The gas parameters before (subscript 1) and after (subscript 2) the discontinuity surface in this case are related by conservation laws, which are conveniently represented as

$$g \equiv \rho_1 U_1 + G' = \rho_2 U_2; \tag{2}$$

$$f \equiv \rho_1 U_1^2 + p_1 + G'U'/F = p_2 + \rho_2 U_2^2; \tag{3}$$

$$h \equiv (C_p T_1 + U_1^2/2)\frac{\rho_1 U_1 F}{\rho_1 U_1 F + G'} + \frac{(C_p' T' + U'^2/2)\,G'}{\rho_1 U_1 F + G'} = \frac{(C_p T_2 + U_2^2/2)\,\rho_2 U_2 F}{\rho_2 U_2 F + G'}; \tag{4}$$

$$e_{V1}^i \rho_1 U_1 + e_V^{i'} G'/F = e_{V2}^i(\rho_2 U_2 + G'/F),\quad C_p = 7R/2\mu, \tag{5}$$

where G' is the flow rate of the gas mixed in; F is the cross-sectional area of the channel at the mixing site; p, ρ, T, and U are the gas pressure, density, temperature, and velocity of the gas;

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e_V^i is the vibrational energy of a unit mass of the i-th component of the mixture $\left(e_V^i = N_i \sum_n \times E_n X_n/\rho\right)$; and E_n is the vibrational energy of the molecule in the state V = n; the prime pertains to the gas mixed in. From (2)-(5) we easily obtain

$$U_2 = \frac{\varkappa f \pm \sqrt{\varkappa^2 f^2 - 2(\varkappa - 1) g^2 h}}{(\varkappa + 1) g}, \quad \rho_2 = \frac{g}{U_2}, \quad T_2 = (h - U_2^2/2)/C_p, \varkappa = 1, 4.$$

The plus sign in the expression for U_2 corresponds to the gas flow up to a mixing surface with $M_1 > 1$ and the minus sign, up to a mixing surface with $M_1 < 1$ (M is the Mach number). The ordinary system of equations of relaxational gas dynamics in the Euler approximation before and after the discontinuity surface [6, 7]:

$$\frac{1}{\rho}\frac{d\rho}{dx} + \frac{1}{U}\frac{dU}{dx} + \frac{1}{F}\frac{dF}{dx} = 0; \tag{6}$$

$$\rho U \frac{dU}{dx} + \frac{dp}{dx} = 0; \tag{7}$$

$$C_p \frac{dT}{dx} + U\frac{dU}{dx} = q_E - \sum_{i=1}^{2} \frac{de_V^i}{dx}; \tag{8}$$

$$U\frac{dX_n(i)}{dx} = Q_V^n(i). \tag{9}$$

Here q_E is the energy source due to the radiation field (q_E, $q_I^n \neq 0$ only in the optical cavity).

The lasing spectrum and the specific energy of the radiation were calculated using the integral condition that the radiation gain and loss be equal in a single pass between mirrors for each lasing transition [8],

$$\left\langle g_{V,j_V}^{V+m,j_V-1} \right\rangle - g_t = 0, \quad V_1 \leqslant V \leqslant V_2, \quad g_t = \frac{1}{2L_c}\ln(R_1 R_2)^{-1},$$
$$\left\langle g_{V,j_V}^{V+m,j_V-1} \right\rangle = \frac{1}{D_c}\int_{x_c}^{x_c+D_c} g_{V,j_V}^{V+m,j_V-1} dx, \tag{10}$$

where g_{V,j_V}^{V+m,j_V-1} is the gain at the middle of the transition line ($V + m$, $j_V - 1 \to V$, j_V) (we considered transitions with m = 1, 2); R_1 and R_2 are the reflection coefficients of the cavity mirrors; L_c is the distance between the mirrors; and D_c is the cavity length along the flow.

For $x_c \leqslant x \leqslant x_c + D_c$

$$q_I^V(i=1) = \frac{g_{V,j_V}^{V+m,j_V-1} I_{V,j_V}^{V+m,j_V-1}}{h\nu_{V,j_V}^{V+m,j_V-1} N_1} - \frac{g_{V-m,j_V}^{V,j_V-1} I_{V-m,j_V}^{V,j_V-1}}{h\nu_{V,j_V}^{V+m,j_V-1} N_1},$$

$$q_E = \sum_{V=V_1}^{V_2} \frac{g_{V,j_V}^{V+m,j_V-1} I_{V,j_V}^{V+m,j_V-1}}{\rho U}.$$

Here ν_{V,j_V}^{V+m,j_V-1} is the frequency and I_{V,j_V}^{V+m,j_V-1} is the radiant intensity of the lasing transition (all told $\ell = V_2 - V_1$). Equations (6)-(9) for $x < x_c$ and (6)-(10) for $x \geqslant x_c$ were integrated numerically as in [8]. The molecular constants necessary for calculating g_{V,j_V}^{V+m,j_V-1} were determined in the same way as in [5] (the values of the collisional widths for transitions with m = 2 were assumed to be equal to the corresponding values for transitions with m = 1). The relations for calculating the rate constants of the VT, VV, and VV' processes in a N_2-DCl mixture were given in [5]. The Einstein coefficients for transitions with m = 1 and 2 were taken from [9].

The specific energy of lasing in the transition ($V + m$, $j_V - 1 \to V$, j_V), $V_1 \leqslant V \leqslant V_2$, is calculated from

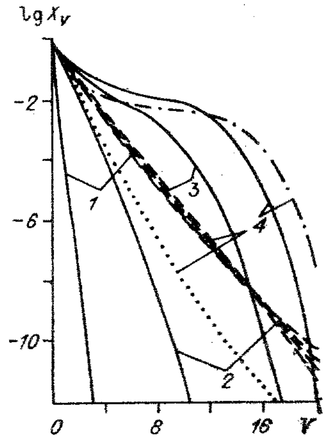

Fig. 1

$$N_e = \sum_{V=V_1}^{V_2} N_{V,j_V}^{V+m,j_V-1}, \quad N_{V,j_V}^{V+m,j_V-1} = \int_{x_c}^{x_c+D_c} g_t I_{V,j_V}^{V+m,j_V-1} dx/\rho_* U_* h_* L_{c*}$$

where ρ_* and U_* are the density and velocity in the critical cross section of the nozzle; and h_* is the height of the critical cross section.

The calculations were carried out for flat nozzles, whose subsonic part is formed by a channel of constant cross section, going over into a wedgelike converging part with a half-angle of 45° and length $4h_*$ ($h_* = 0.2$ mm), and the contour of the supersonic part is described by the exponential function

$$F(x) = 1 + (F_k - 1)[1 - \exp(-2x/(F_k - 1))]$$

(F_k is the relative height of the channel in the nozzle exit $x = x/h_*$). The mixing DCl molecules (T = 300 K) into the flow of preheated N_2 was simulated by introducing a mixing surface into the supersonic part of the nozzle at a distance of $5h_*$ from the critical cross section.

Figure 1 illustrates how distribution function of N_2 and DCl molecules (dashed and solid lines) over the vibrational levels varies with the distance x from the critical cross section of a nozzle with $F_k = 500$ after 4% DCl is mixed into N_2 with the initial parameters $T_0 = 2500$ K and $p_0 = 10$ MPa. Lines 1-4 correspond to $x = 5h_*$, $15h_*$, $1100h_*$, and $1500h_*$ (the last cross section coincides with the nozzle exit section). For comparison dash-dot and dashed lines show the distributions X_V for DCl and N_2 in the cross section $x = 1500h_*$ in an ordinary gas-dynamic laser with the same parameters but for $\gamma_1 = 0.07$ (g_{V,j_V}^{V+1,j_V-1} have maximum values for this γ_1). With increasing distance from the mixing site the distribution function of the DCl molecules among the vibrational levels differs increasingly from the Boltzmann distribution (for x = 5) and for x = 1500 takes on a characteristic shape with a plateau in the region $4 \leqslant V \leqslant 12$. Such a distribution of DCl molecules forms because of energy transfer during non-resonant VV' exchange with excitation of N_2 molecules. The distribution of N_2 molecules among the vibrational levels also changes. The distribution $N_2(V)$ differs most from the Boltzmann distribution in the region $3 \leqslant V \leqslant 14$ and has a characteristic sag. In an ordinary N_2-DCl gasdynamic laser the formation of a "plateau" in the distribution X_V is due mainly to intramode VV exchange and VT relaxation [5]. In a GDL with mixing the population of the DCl levels in the region of the plateau is higher than in an ordinary GDL. The population of the N_2 levels is also higher in this case. The explanation for this is that without DCl molecules in the subsonic and transonic parts of the nozzle the relaxation of the energy of N_2 molecules is much slower and a large part of the energy remains in the vibrations of N_2 and DCl molecules to the nozzle exit. The considerable population of the vibrational levels of DCl in the region of the plateau causes inversion not only in the $(V + 1, j_V) \to (V, j_V - 1)$ transitions but also in the $(V + 2, j_V) \to (V, j_V - 1)$ harmonics. This is illustrated by Fig. 2, which shows the gain obtained in the fundamental transitions (a) and in harmonics (b) in the nozzle exit for the above parameters. Here and below the notation for the gain and lasing lines is given according to the values of V and j_V of the lower state. The values of g_{V,j_V}^{V+2,j_V-1} are roughly one-tenth those of g_{V,j_V}^{V+1,j_V-1}, but are sufficient for lasing at values of g_t ($g_t \leqslant 0.01$ m^{-1}) typical of cavities used at present. In an ordinary GDL with the

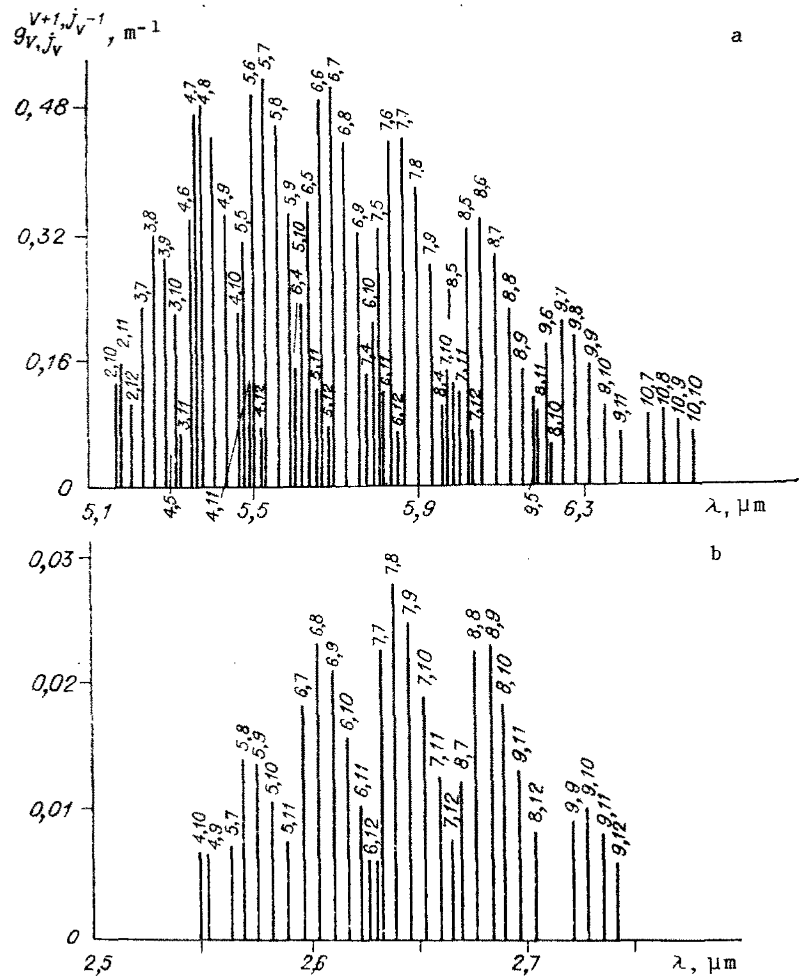

Fig. 2

same parameters g_{V,j_V}^{V+1,j_V-1} is almost 1.5 times higher. This is because mixing DCl into the supersonic N_2 flow raises the gas temperature, which is higher in the nozzle exit than during expansion of the N_2−DCl mixture. Even though the population of the DCl vibrational levels is higher than in a GDL based on mixing, the inversion in the given vibrational-rotational transition is smaller because of the high population of the rotational half-level of the lower state of the amplifying transition.

Let us now consider how the amount of DCl mixed into the N_2 flow (T_0 = 2500 K, p_0 = 10 MPa) affects the lasing spectrum and the specific radiant energy. Figure 3 shows the spectra of lasing at the fundamental frequencies N_{V,j_V}^{V+1,j_V-1} (λ) (a) and at harmonics N_{V,j_V}^{V+2,j_V-1} (λ) (b), which occurs when 2, 4, and 6% DCl (dashed, solid, and dash-dot lines) is mixed into N_2 in a nozzle with ε = 500 for D_c = 2500h_* and g_t = 0.01 m^{-1}. In all cases the cavity began at the nozzle exit. The maximum values of N_{V,j_V}^{V+m,j_V-1} ($m = 1, 2$) are reached under these conditions when 4% DCl is mixed in. An interesting feature here is the fact that the maximum values of N_{V,j_V}^{V+1,j_V-1} are obtained in transitions with small V (2 → 1, 3 → 2, 1 → 0); in lasing at the harmonics (λ = 2.5-2.8 μm) the maxima of N_{V,j_V}^{V+2,j_V-1} correspond to V = 5, 6, 7, while the values of j_V, conversely, are lower. As in an ordinary gasdynamic laser, the lasing spectrum shifts to higher wavelengths if the DCl content in the mixture increases. The value of N_e is ~42.2 J/g in lasing at the fundamental frequencies and ~6.7 J/g in lasing at harmonics.

One of the main advantages of lasers with mixing of N_2 and DCl in comparison with ordinary GDLs is that the temperature in the heating chamber can be raised to T_0 > 2500 K, since in the given case DCl does not dissociate (this limits the value of T_0 in an ordinary GDL). The fraction of energy stored in N_2 vibrations increases and as a result N_e should increase. On the other hand, other parameters being equal, the gas temperature beyond the nozzle exit

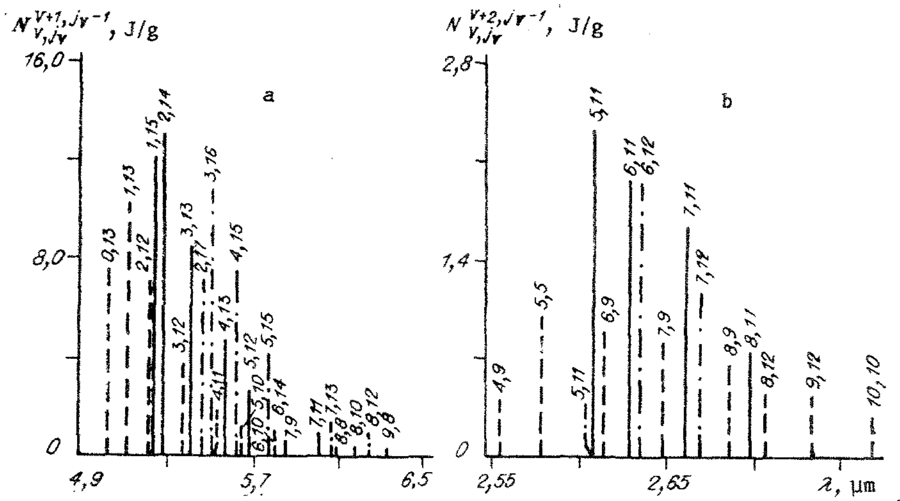

Fig. 3

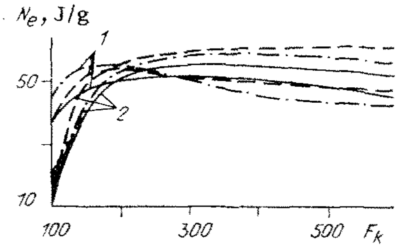

Fig. 4

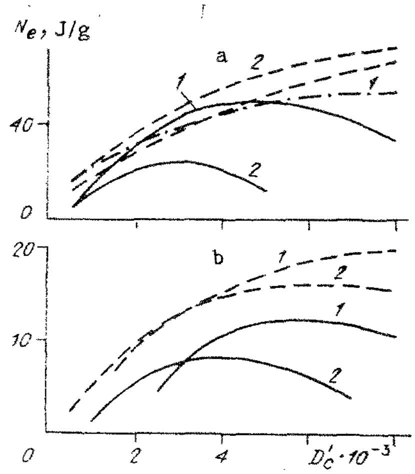

Fig. 5

also rises and so N_e should decrease. One more factor causing N_e to change at a fixed cavity length along the flow is the decrease in the exchange length between N_2 and DCl because the density of molecules decreases. The dependence of N_e on T_0, therefore, is rather complicated in nature, which varies as a function of p_0, γ_1, and F_k. Figure 4 shows the $N_e(F_k)$ curves for lasing in transitions with m = 1 in a cavity with $g_t = 0.01\ m^{-1}$ and $D_c = 5000h_*$ for T_0 = 2000, 2500, and 3000 K (solid, dashed, and dash-dot curves), p_0 = 10 MPa, and γ_1 = 0.02 and 0.04 (lines 1, 2). A rise in T_0 for $F_k < 200$ when $\gamma_1 = 0.02$ causes N_e to increase. Since an increase in T_0 from 2500 to 3000 K for F_k = 100 causes N_e to increase by 27%. At the same time at $\gamma_1 = 0.04$ an increase in T_0 from 2500 to 3000 K does not cause N_e to grow for any F_k. Moreover, for $F_k > 350$ an increase in T_0 when $\gamma_1 = 0.02$ can even result in a lower N_e.

The radiant energy of a GDL with mixing of N_2 and DCl depends not only on the initial gas parameters and the amount of DCl mixed in but also on the geometric characteristics, such as the nozzle expansion ratio and the cavity length along the flow. Figure 5 shows the curves of $N_e(D_c')$, $D_c' = D_c/h_*$, obtained when calculating lasing in the fundamental transitions (a) and in harmonics (b) for γ_1 = 0.02 and 0.04 (lines 1, 2), T_0 = 2500 K, p_0 = 10 MPa, g_t = 0.01 m^{-1} for various F_k (for transitions with m = 1 the solid and dashed curves correspond to F_k = 100 and 300 and those with m = 2 correspond to F_k = 300 and 500, while for F_k = 100 lasing does not occur in these transitions). The dash-dot lines here show $N_e(D_c')$ for an ordinary GDL with the same values of T_0, p_0, g_t, but with $\gamma_1 = 0.07$ (this composition corresponds to maximum values of N_e for F_k = 300). We see that an increase in D_c' up to the optimum value D_c^{opt} results in the growth of N_e (for F_k = 300 for m = 1 and F_k = 500 for m = 2 the optimum value D_c^{opt} is reached for $D_c > 10^4$). The value of D_c^{opt} increases as F_k grows and more DCl is mixed in.

The explanation for the existence of an optimum D_c is that for efficient extraction of energy from the N_2 vibrations the cavity length should be greater than the length of the VV' exchange between N_2 and DCl molecules but smaller than the VT relaxation length. We also see that even for moderate $D_c = 5000h_*$ (1 m) the value of N_e in a mixing laser lasing in the fundamental transitions is 25% higher than in an ordinary GDL and may reach 62 J/g for F_k = 300. With increasing D_c this difference becomes even larger. At the same time for small $D_c = 500h_*$ the value of N_e is the same in both types of laser. This is because for a small D_c only a small fraction of the energy stored in the N_2 vibrations is converted into the energy of coherent radiation. In a mixing GDL operating on transitions with m = 2 a value $N_e \geqslant 10$ J/g can be obtained even for $D_c \geqslant 2000h_*$.

In summary, our analysis shows that a gasdynamic laser with mixing of N_2 and DCl can be a very efficient device for obtaining radiation with λ = 5-7 μm (fundamental-frequency transitions) and with λ = 2.5-2.8 μm (harmonics). The specific radiant energy may reach 70 and 20 J/g, respectively, in systems of moderate size.

LITERATURE CITED

1. Advanced H_2–HCl Gas Dynamic Laser, N ARC–47–5655, p. 1, Rept. Atlantic Res. Corp., Alexandria, VA (1976).
2. A. N. Oraevskii, N. B. Rodionov, and V. A. Shcheglov, "Thermal gasdynamic laser with partial population inversion," Zh. Tekh. Fiz., 48, No. 7 (1978).
3. V. A. Levin and A. M. Starik, "Analysis of hydrogen halide lasers," Kvantovaya Élektron. (Moscow), 9, No. 2 (1982).
4. N. G. Dautov and A. M. Starik, "Numerical analysis of characteristics of N_2–DCl gas-flow lasers," Teplofiz. Vys. Temp., 26, No. 1 (1992).
5. N. G. Dautov and A. M. Starik, "Numerical analysis of energy and spectral characteristics of N_2–DCl gasdynamic laser," Khim. Fiz., 12, No. 4 (1993).
6. R. I. Solukhin and N. A. Fomin, Mixing Gasdynamic Lasers [in Russian], Nauka i Tekhnika, Minsk (1984).
7. K. Smith and R. M. Thompson, Computer Modeling of Gas Lasers, Plenum, New York (1978).
8. V. A. Vostryakov, I. P. Kirmusov, and A. M. Starik, "Contribution to the calculation of multiple-frequency lasing of diatomic molecular gasdynamic lasers," Khim. Fiz., 7, No. 4 (1988).
9. J. M. Herbelin and G. Emmanuel, "Einstein coefficients for diatomic molecules," J. Chem. Phys., 60, No. 2 (1974).

NUMERICAL ANALYSIS OF A PLASMA JET IN A MAGNETIC FIELD

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The behavior of supersonic plasma jets in an external magnetic field is of interest for problems involving the use of plasma accelerators of various types in space technology. The interplay of the jet and the magnetic field must be known if such problems are to be solved. The complexity of the pertinent experiments stimulates the use of computer simulation to obtain fuller theoretical concepts about the nature of the behavior of plasma jets. Comparatively few theoretical studies on plasma formations in a magnetic field have dealt directly with supersonic plasma jets. We note that in [1] Savel'ev used the one-temperature MHD approximation for a numerical analysis of a plane plasma jet bounded in the transverse direction.

In the work reported here, within the framework of the two-temperature MHD model we have considered the behavior of a highly underexpanded (nearly a vacuum type) supersonic plasma jet with a superimposed magnetic field, taking the induced magnetic field into account. We have studied the effect of the magnetic field on the geometry of the jet boundary, the nature of the flow, the distribution of parameters in the jet, and the perturbation of the external magnetic field by the electric currents of the jet.

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